(8 pages) **Reg. No. :** .....

Code No. : 7872 **Sub. Code : VMAC 11**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2025.

First Semester

Mathematics – Core

GROUP THEORY

(For those who joined in July 2024 and 2025 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($15 \times 1 = 15$ marks)

Answer ALL questions.

Choose the correct answer :

1. If G is a group and H is a subgroup of index 2 in G then

   (a) H is a normal subgroup of G

   (b) H is abelian in G

   (c) $aHa^{-1} \neq H$ for all $a \in G$

   (d) $aHa^{-1} = H$ for all $a \in G$

2. If $\phi : G \to \overline{G}$ is homomorphism then which is false?

   (a) $\phi(ab) = \phi(a)\phi(b)$ for all $a, b \in G$

   (b) $[\phi(x)]^{-1} = \phi(x^{-1})$ for $x \in G$

   (c) $\phi(e) = \bar{e}$, unit element of $\overline{G}$

   (d) None of these

3. Every subgroup of an abelian group is

   (a) abelian (b) an ideal

   (c) normal (d) none of these

4. Let G be a group and ϕ be an automorphism of G. If $a \in G$ is of order $O(a) > 0$ then

   (a) $O(a) = O(\phi(a))$

   (b) $O(a) \neq O(\phi(a))$

   (c) $O(\phi(a)) > O(a)$

   (d) None

5. If $a \in Z(G)$ then

   (a) $N(a) = G$ (b) $N(a) \neq G$

   (c) $O(z) = p^a$ (d) None of these

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6. Let G be a group of order 36 and G has a subgroup H of order 9. Then $i(H) =$

(a) 4 (b) 3

(c) 5 (d) 1

7. Let A_n be the set of all even permutation in S_n. Then $O(A_n) =$

(a) $n!$ (b) $\frac{n!}{2}$

(c) n (d) $\frac{n}{2}$

8. If $O(G) = p^2$ (p is prime) then G is

(a) cyclic (b) abelian

(c) non-abelian (d) none of these

9. If $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ then $\alpha\beta =$

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

10. Let G be a group of order 20449. The number of 11-sylow subgroup of G is

(a) 0 (b) 1

(c) 11 (d) none of these

11. S_p^k has a p-sylow subgroup of order

(a) $p^{n(k)}$ (b) p

(c) p^k (d) none of these

12. Number of p-sylow subgroup in G is of the form

(a) $1+k_p$ (b) $1-k_p$

(c) $p+k$ (d) $p-k+1$

13. Every finite abelian group is the direct product of

(a) subgroup (b) abelian group

(c) cyclic group (d) none of these

14. If A and B are group then $A \times B$ is isomorphic to

(a) A (b) B

(c) A and B (d) none of these

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15. Number of non-isomorphic abelian group of order 16 is

(a) 4 (b) 5

(c) 3 (d) 2

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If G is a finite group and N is a normal subgroup of G then $O\left(\frac{G}{N}\right)=\frac{O(G)}{O(N)}$.

Or

(b) If $\phi: G \to \overline{G}$ is homomorphism then

(i) $\phi(e)=\bar{e}$, the unit element of $\overline{G}$

(ii) $[\phi(x)]^{-1}=\phi(x)\ \forall x \in G$.

17. (a) Let G be a group and ϕ be an automorphism of G. If $a \in G$ is of order $O(a)>0$ then $O(\phi(a))=O(a)$.

Or

(b) Define solvable group. Prove that a subgroup of a solvable group is solvable.

18. (a) Prove that conjugacy is an equivalence relation.

Or

(b) State and prove Cauchy's theorem.

19. (a) Prove that $n(k)=1+p+p^2+....+p^{k-1}$.

Or

(b) If A and B are finite subgroup of G then $O(AxB)=\frac{O(A)O(B)}{O(A \cap xBx^{-1})}$.

20. (a) Let G be a group and G be the internal direct product of $N_1, N_2, ..., N_n$. Let $T=N_1 \times N_2 \times ... \times N_n$. Then G is isomorphic to T.

Or

(b) If G and G' are isomorphic abelian groups then for all integer p, $G(s)$ and $G'(s)$ are isomorphic.

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PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) If H and K are finite subgroups of G of orders $O(H)$ and $O(K)$ then $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$.

Or

(b) State and prove Sylow's theorem for abelian groups.

22. (a) State and prove Cayley's theorem.

Or

(b) (i) G is solvable if and only if $G^{(k)} = (e)$ for $k \in Z$.

(ii) If G is a solvable group and $\overline{G}$ is a homomorphic image of G then $\overline{G}$ is solvable.

23. (a) If G is a finite group then $C_a = O(G)/O(N(a))$.

Or

(b) (i) Prove that $N(a)$ is a subgroup of G.

(ii) Prove that $a \in Z$ if and only if $N(a) = G$. If G is finite, $a \in Z$ if and only if $O(N(a)) = O(G)$.

24. (a) State and prove second part of Sylow's theorem.

Or

(b) Prove that S_{p^k} has a p-sylow subgroup.

25. (a) Prove that every finite abelian group is a direct product of cyclic gruops.

Or

(b) Suppose that G is the internal direct product of $N_1, N_2, \ldots, N_n$. Then for $i \neq j, N_i \cap N_j = (e)$ and if $a \in N_i, b \in N_j$ then $ab = ba$.

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Reg. No. : .....

Code No. : 7389 **Sub. Code : WMAM 12/ VMAC 12**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2025

First Semester

Mathematics — Core

REAL ANALYSIS — I

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($15 \times 1 = 15$ marks)

Answer ALL questions.

Choose the correct answer :

1. A function $f(x)$ is said to be monotonic if

   (a) It is either increasing or decreasing but not necessarily consistently

   (b) It never increases and never decreases

   (c) It is either strictly increasing or strictly decreasing over its entire domain

   (d) It has exactly one local maximum and one local minimum

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2. A function f of bounded variation on an interval $[a,b]$?

(a) f must be linear on $[a,b]$

(b) f can be decomposed into the difference of two monotonic functions

(c) f must be differentiable on $[a,b]$

(d) f is necessarily periodic

3. Which of the following tests can be used to determine if a series converges absolutely?

(a) Ratio Test (b) Comparison Test

(c) Integral Test (d) All of the above

4. For a change of variable $x = g(t)$ in the Riemann-Stieltjes integral $\int_a^b f(x)\,d\alpha(x)$, which condition must $g(t)$ satisfy for the integral transformation to be valid?

(a) $g(t)$ must be a differentiable function with $g'(t) \neq 0$ over the interval

(b) $g(t)$ must be a linear function

(c) $g(t)$ must be an increasing function

(d) $g(t)$ must be a continuous function

5. What is Riemann's Condition for a function $\alpha(x)$ with respect to the Riemann-Stieltjes integral $\int_a^b f(x)\,d\alpha(x)$?

(a) $\alpha(x)$ must be differentiable on $[a,b]$

(b) $\alpha(x)$ must be of bounded variation on $[a,b]$

(c) $\alpha(x)$ must be continuous on $[a,b]$

(d) $\alpha(x)$ must be strictly increasing on $[a,b]$

6. If $\alpha(x)$ and $\beta(x)$ are two functions of bounded variation on $[a,b]$ and c is a constant, then $\int_a^b f(x)\,d(c\alpha(x)) =$

(a) $c\int_a^b f(x)\,d\alpha(x)$

(b) $\int_a^b f(x)\,d\alpha(x) + c\int_a^b f(x)\,d\alpha(x)$

(c) $\int_a^b cf(x)\,d\alpha(x)$

(d) $\int_a^b f(x)\,d\alpha(x)\cdot c$

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7. For the Mean Value Theorem for the Riemann-Stieltjes integral $\int_a^b f(x)\,d\alpha(x)$, which condition must the function $\alpha(x)$ satisfy?

(a) $\alpha(x)$ must be differentiable on $[a,b]$ $[a,b]$ $[a,b]$

(b) $\alpha(x)$ must be a non-decreasing function of bounded variation

(c) $\alpha(x)$ must be continuous and strictly increasing

(d) $\alpha(x)$ must be a constant function

8. If $f(x)$ is continuous on $[a,b]$ and $\alpha(x)$ is differentiable, what does the Second Fundamental Theorem of Integral Calculus imply about the derivative of the integral $\int_a^x f(t)d\alpha(t)$?

(a) The derivative is $\alpha'(x)\cdot f(x)$

(b) The derivative is $f(x)\cdot\alpha(x)$

(c) The derivative is $f(x)\cdot\alpha'(x)$

(d) The derivative is $f'(x)\cdot\alpha(x)$

9. Which of the following is not a necessary condition for the existence of the Riemann-Stieltjes integral $\int_a^b f(x)\,d\alpha(x)$?

(a) $f(x)$ must be bounded on $[a,b]$

(b) $\alpha(x)$ must be a function of bounded variation

(c) $\alpha(x)$ must be continuous on $[a,b]$

(d) $f(x)$ must be integrable on $[a,b]$

10. Which of the following is an example of a condition that ensures the convergence of a double sequence $\{a_{m,n}\}$ to a limit L?

(a) $\{a_{m,n}\}$ is eventually constant

(b) $\{a_{m,n}\}$ is uniformly bounded and every subsequence converges to L

(c) For every $\varepsilon>0$, there exists N such that $|a_{m,n}-L|<\varepsilon$ for all $m,n\geq N$

(d) $\{a_{m,n}\}$ is bounded and convergent in a normed space

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11. The necessary condition for the Cauchy product of Σa_n and Σb_n to be convergent is

(a) at least one of the series Σa_n or Σb_n must be absolutely convergent

(b) Both series Σa_n and Σb_n must be conditionally convergent

(c) Both series Σa_n and Σb_n must be divergent

(d) Both series Σa_n and Σb_n must be convergent but not necessarily absolutely convergent

12. What is the relationship between the radius of convergence of the product of two power series and their individual radii of convergence?

(a) The radius of convergence of the product series is the product of the individual radii

(b) The radius of convergence of the product series is the sum of the individual radii

(c) The radius of convergence of the product series is the minimum of the individual radii

(d) The radius of convergence of the product series is the maximum of the individual radii

13. The sufficient condition for the uniform convergence of a series of functions $\sum f_n(x)$ on a domain D?

(a) The series $\sum f_n(x)$ converges pointwise on D

(b) The series $\sum f_n(x)$ converges absolutely on D

(c) The series $\sum f_n(x)$ is uniformly bounded on D

(d) The series $\sum f_n(x)$ converges uniformly on D

14. Which of the following statements is true for convergence in the mean?

(a) Convergence in the mean implies uniform convergence

(b) Convergence in the mean is equivalent to pointwise convergence

(c) Convergence in the mean implies convergence in measure

(d) Convergence in the mean does not necessarily imply convergence in L^2

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15. If $\{f_n\}$ converges uniformly to f on $[a,b]$ and f_n are differentiable with f_n' converging uniformly to g, then which of the following is true?

(a) f_n converges uniformly to f and f is differentiable with f' = g

(b) f_n converges pointwise to f but f is not necessarily differentiable

(c) f_n converges uniformly to f and f is not differentiable

(d) The convergence of f_n to f implies the convergence of f_n' to g

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that if Σa_n is a series of complex terms whose partial sums form a bounded sequence and $\{b_n\}$ is a decreasing sequence which converges to 0, then $\Sigma a_n b_n$ converges.

Or

(b) Prove that if Σa_n is conditionally convergent then both Σp_n and Σq_n diverges where $p_n = \frac{|a_n| + a_n}{2}$ and $q_n = \frac{|a_n| - a_n}{2} (n = 1,2,\ldots)$.

17. (a) Prove that if $f \in R(\alpha)$ on $[a,b]$, then $\alpha \in R(f)$ on $[a,b]$ and $\int_a^b f(x)d\alpha(x) + \int_a^b \alpha(x)df(x) = f(b)\alpha(b) - f(a)\alpha(a)$.

Or

(b) Prove that if $\alpha \nearrow$ on $[a,b]$, then $\bar{I}(f\alpha) \leq \underline{I}(f,\alpha) + \epsilon$.

18. (a) State and prove second mean-value theorem for Riemann-Stieltjes integrals.

Or

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(b) Prove that if $f \in R$ and $g \in R$ on $[a,b], F(x) = \int_a^x f(t)\,dt, G(x) = \int_a^x g(t)\,dt$ for $x \in [a,b]$ then F and G are continuous functions of bounded variation on $[a,b]$. And also prove that for $f \in R(G)$ and $g \in R(F)$ on $[a,b]$ then $\int_a^b f(x)g(x)dx = \int_a^b f(x)dG(x) = \int_a^b g(x)dF(x)$.

19. (a) Prove that for a fixed point p if $\lim_{p,q\to\infty} f(p,q)$ exists and $\lim_{p,q\to\infty} f(p,q) = a$ then the limit $\lim_{p\to\infty}\left(\lim_{q\to\infty} f(p,q)\right)$ so exists and has the value a.

Or

(b) Prove that for k^{th} prime number, p_k if $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \prod_{k=1}^{\infty} \frac{1}{1-p_k^{-s}}$ for $s > 1$, then the product converges absolutely.

20. (a) Prove that if each f_n is continuous at a point c of S, then the limit function f is also continuous at c, where $f_n \to f$. f uniformly on S.

Or

(b) State and prove Cauchy condition for uniform convergence of series.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) Prove the for a bounded variation f on $[a,b]$ with $x \in (a,b]$ and $V(x) = V_f(a,x)$ and $V(a) = 0$, then every point of continuity of f is also a point of continuity of V and also prove that the converse is true.

Or

(b) Prove that if Σa_n is an absolutely convergent series having sum s, then every rearrangement of Σa_n also converges absolutely and has sum s.

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22. (a) Prove that for $h \in R(\beta)$ on S and $\int_a^b f\, d\alpha = \int_c^d h\, d\beta$ then $\int_{g(c)}^{g(d)} f(t)\, d\alpha(t) = \int_c^d f[g(x)]d\{\alpha[g(x)]\}$ where $f \in R(\alpha), g$ is strictly monontonic continuous function defined on an interval S having endpoints c and $d, a = g(c), b = g(d), h$ and β are the composite functions defined as $h(x) = f[g(x)], \beta(x) = \alpha[g(x)]$, if $x \in S$.

Or

(b) Prove that for $\alpha \nearrow$ on $[a,b]$, the following statements are equivalent

(i) $f \in R(\alpha)$ on $[a,b]$

(ii) f satisfies Riemann's condition with respect to α on $[a,b]$.

(iii) $\underline{I(f,\alpha)} = \underline{I}(f,\alpha)$.

23. (a) State and prove change of variable in a Riemann integral.

Or

(b) State and prove second fundamental theorem of integral calculus.

24. (a) Prove that the infinite product $\prod u_n$ converges if and only if for every $\epsilon > 0$ there exists an N such that $n > N$ implies $|u_{n+1} y_{n+2} .. u_{n+k} - 1| < \epsilon$, for $k = 1,2,3....$

Or

(b) Prove that for each z in $B(z_0,r) \Sigma a_n (z - z_0)^n$ converges then the function f defined by the equation $f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$ if $z \in B(z_0;r)$ has a derivative $f'(z)$ for each z in $B(z_0;r)$ given by $f'(z) = \sum_{n=1}^{\infty} n a_n (z - z_0)^{n-1}$.

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25. (a) Prove that if each term of $\{f_n\}$ is a real-vlaued function having a finite derivative at each point of an open interval (a,b), for at least one point x_0 in (a,b) then there exists a function f such that $f_n \to f$ uniformly on (a,b) and for each x in (a,b) the derivative $f'(x)$ exists and equals $g(x)$.

Or

(b) Prove that $\lim_{n\to\infty} \int_a^b f_n(t)\,dt = \int_a^b \lim_{n\to\infty} f_n(t)\,dt = \int_a^b f(t)\,dt$ where $f_n \in R$ on $[a,b]$ is a boundedly convergent sequence on $[a,b]$, limit function $f \in R$ on $[a,b]$ and $P = \{x_0, x_1, \ldots, x_m\}$ is a partition P of $[a,b]$ such that on every subinterval $[c,d]$ not containing any of the points x_k, the sequence $\{f_n\}$ converges uniformly to f.

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Code No. : 7390 **Sub. Code : WMAM 13/ VMAC 13**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2025.

First Semester

Mathematics — Core

ORDINARY DIFFERENTIAL EQUATIONS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. The solution of $y'' + y' - 2y = 0$ is

(a) $c_1e^{x} + c_2e^{-2x}$ (b) $c_1e^{-x} + c_2e^{-2x}$

(c) $c_1e^{-x} + c_2e^{2x}$ (d) $c_1e^{x} + c_2e^{2x}$

2. Two functions g_1, g_2 defined on an interval I are said to be _____ on I if there exists two constants C_1, C_2 not both zero such that $C_1g_1(x) + C_2g_2(x) = 0$ for all x in I.

(a) Linearly dependent

(b) Linearly independent

(c) Both (a) and (b)

(d) Neither (a) nor (b)

3. If $\phi_1 = x$ and $\phi_2 = x^3$ then the value of the Wronskian is _____.

(a) $3x^3$ (b) x^3

(c) $2x^3$ (d) $4x^3$

4. The characteristic polynomial of $y'' + w^2y = 0$ is _____.

(a) $r^2 + w^2 = 0$ (b) $r^2 + r = 0$

(c) $r^2 + w^2r = 0$ (d) $r^2 + w^2 + r = 0$

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5. If $\phi_1,\ldots,\phi_n$ are n solutions of $L(y)=0$ on an interval I, they are _____ if $w(\phi_1,\phi_2,\ldots,\phi_n)(x)\neq 0$ for all x in I.

(a) Linearly dependent

(b) Linearly independent

(c) Real numbers

(d) Integers

6. The solution of $y'''+y''+y'+y=1$ is

(a) -1 (b) 0

(c) 1 (d) no solution

7. The value of $P_n(-1)=$_____.

(a) -1 (b) 1

(c) $(-1)^n$ (d) 0

8. The equation $y''-2xy'+2\alpha y=0$, where α is a constant, is called the _____.

(a) Legendre equation (b) Bessel's equation

(c) Chebyshev equation (d) Hermite equation

9. The value of the Legendre Polynomial $P_2(x)$ is _____.

(a) $\frac{3}{2}x^2-\frac{1}{2}$ (b) x

(c) x^2 (d) 1

10. The singular point and its nature of the equation $x^2y''-5y'+3x^2y=0$ is

(a) $x=0$; regular (b) $x=1$; regular

(c) $x=0$; not regular (d) $x=1$; irregular

11. The series J_0 is _____ for $(x)<\infty$.

(a) Divergent (b) Continuous

(c) Equal (d) Convergent

12. $\lceil n+1=$ _____.

(a) n (b) $n!$

(c) $n\cdot n!$ (d) $(n+1)!$

13. The Lipschitz constant for the function $f(x,y)=4x^2+y^2$, on $S:|x|\leq 1,\ |y|\leq 1$ is _____.

(a) 2 (b) 3

(c) 1 (d) -1

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14. If $y' = 1 - 2xy, y(0) = 0$ then $\phi(x) =$ _____.

(a) $e^x \int e^{t^2} dt$ (b) $e^{-x^2} \int e^{t^2} dt$

(c) $e^{-x^2} \int e^t dt$ (d) $e^{x^2} \int e^t dt$

15. The equation $(x^2 + xy)dx + xy\,dy = 0$ is _____.

(a) exact (b) not exact

(c) both (a) and (b) (d) neither (a) nor (b)

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

16. (a) Verify whether the functions ϕ_1, ϕ_2 given by $\phi_1(x) = \sin x$, $\phi_2(x) = e^{ix}$ are linearly independent or not.

Or

(b) If ϕ_1, ϕ_2 are two solutions of $L(y) = 0$ on an interval I containing a point x_0 then prove that $W(\phi_1, \phi_2)(x) = e^{-a_1(x-x_0)} W(\phi_1, \phi_2)(x_0)$.

17. (a) Solve $y''' - 3y' + 2y = 0$.

Or

(b) Let $\alpha_1, \ldots, \alpha_n$ be any n constants and let x_0 be any real number. Prove that there exists a solution ϕ of $L(y) = 0$ on $-\infty < x < \infty$ satisfying $\phi(x_0) = \alpha_1, \phi'(x_0) = \alpha_2, \ldots, \phi^{(n-1)}(x_0) = \alpha_n$.

18. (a) Prove that if $\phi_1, \phi_2, \ldots, \phi_n$ are n-solutions of $L(y) = 0$ on an interval I they are linearly independent $\Leftrightarrow W(\phi_1, \ldots, \phi_n)(x) \neq 0 \; \forall x \in I$.

Or

(b) Find all the solution of the equation $y'' - \frac{2}{x^2} y = x, \; 0 < x < \infty$.

19. (a) Find two linearly independent solution of the equation $x^2 y'' - 2x(x+1)y' + 2(x+1)y = 0$.

Or

(b) Prove that $x^{1/2} J_{-1/2}(x) = \frac{\sqrt{2}}{\Gamma(1/2)} \cos x$.

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20. (a) Find all the real valued solutions $y' = x^2 y$.

Or

(b) Solve the initial value problem $y' = xy, y(0) = 1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

21. (a) Find all the solutions of the equation $y'' + 9y = \sin 3x$.

Or

(b) Solve the equation
$y'' + y = \sec x \left(-\pi/2 < x < \pi/2\right)$.

22. (a) Let ϕ be any solution of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I containing a point x_0. Then prove that for all x in I.
$\|\phi(x_0)\| e^{-x|x-x_0|} \le \|\phi(x)\| \le \|\phi(x_0)\| e^{k|x-x_0|}$ where
$\|\phi(x)\| = \left[\left|\phi(x)^2 + \phi'(x)^2\right|\right]^{1/2}$, $K = 1 + |\alpha_1| + |\alpha_2|$.

Or

(b) Solve $L(y) = y'' - 3y' + 2y = x^2$.

23. (a) Solve the Legendre equation :
$L(y) = (1-x)^2 y'' - 2xy' + \alpha(\alpha+1)y = 0$.

Or

(b) Find two linearly independent power series solutions of the equation $y'' + y = 0$.

24. (a) Find the two independent solutions of second order Euler equation $x^2 y'' + axy' + by = 0$.

Or

(b) Find the solution ϕ of the form $\phi(x) = x^r \sum_{k=0}^{\infty} C_k x^k$, $(x > 0)$ for the equation $2x^2 y'' + \left(x^2 - x\right)y' + y = 0$.

25. (a) Show that A function ϕ is a solution of the initial value problem $y' = f(x, y), y(x_0) = y_0$ on an interval $I \Leftrightarrow$ it is a solution of the integral equation $y = y_0 + \int_{x_0}^{x} f(t, y)\, dt$ on I.

Or

(b) Write the Existence theorem for convergence of the successive approximations and prove it.

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Code No. : 7391 **Sub. Code : WMAE 11/ VMAE 11**

M.Sc.(CBCS) DEGREE EXAMINATION, NOVEMBER 2025.

First Semester

Mathematics – Elective I

GRAPH THEORY AND APPLICATIONS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. A simple graph G is called _____ if $G \cong G^c$

(a) Self- complementary

(b) Non Adjacent

(c) Disconnected

(d) Isomorphic

2. If a graph G has property P, then G cannot contain an induced subgraph isomorphic to H. Such graph H is called _____

(a) Forbidden subgraph (b) Line graph

(c) Graph operation (d) Union of graph

3. A _____ of G is a complete subgraph of G.

(a) Induced subgraph (b) Proper subgraph

(c) Clique (d) Spanning subgraph

4. A graph G is r — *connected* if

(a) $k(G) \geq r$ (b) $k(G) > r$

(c) $k(G) < r$ (d) $k(G) \leq r$

5. A subset V^1 of the vertex set $V(G)$ of a connected graph G is a _____ of G if G — V' disconnected graph

(a) vertex cut (b) edge cut

(c) cut edge (d) cut vertex

6. A _____ of a graph is a maximal nonseparable sub graph of G.

(a) blocks (b) hypothesis

(c) network (d) walk

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7. A _____ at a vertex u of a tree T is a maximal subtree containing u as an end vertex.

(a) Branch (b) Loop

(c) Contraction (d) Tree

8. Let G be a connected graph. Then the diameter of G is defined as _____ .

(a) $\max\{d(u,v : u,v \in V(G)\}$

(b) $\min\{d(u,v : u,v \in V(G)\}$

(c) $\max\{d(u,v : u,v \notin V(G)\}$

(d) $\min\{d(u,v : u,v \notin V(G)\}$

9. A vertex V *of* G *is called a* _____ *if* $e(v) = r(G)$.

(a) Diameter (b) Radius

(c) Central vertex (d) Eccentricity

10. As subset S of the vertex set V of a graph G is called _____ if no two vertices of S are adjacent in G.

(a) Independent (b) Matching

(c) Minimum (d) Covering

11. A _____ in G is a set of independent edges.

(a) Matching (b) Edge covering

(c) Independent (d) Maximum

12. A matching M of a graph G is maximum if and only if G has no _____ path.

(a) M-augmenting (b) Factor

(c) Matching (d) Independent

13. For a graph to have an Eulerian path but not necessarily an Eulerian circuit, it must

(a) Be connected and have exactly two vertices with odd degrees.

(b) Be connected and have exactly one vertex with odd degrees.

(c) Be disconnected and have all vertices with even degrees.

(d) Be connected and have all vertices with even degrees.

14. A graph is said to be Hamiltonian if.

(a) It contains a Hamiltonian cycle

(b) It contains a Hamiltonian path

(c) It is bipartite

(d) It contains an Eulerian path

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15. Vertex coloring of a graph is defined as:

(a) An assigment of colors to edges such that no two adjacent edges have the same color

(b) An assigment of colors to vertices such that no two adjacent vertices have the same color.

(c) An assigment of colors to vertices such that all verticals have the same color.

(d) an assigment of color to edges such as that no two edges that share a common vetex have the same color.

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 250 words.

16. (a) Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of its edges.

Or

(b) Prove that the number of edges of a simple graph of order n having w components cannot exceed $\frac{(n-w)(n-w+1)}{2}$.

17. (a) Show that an edge $e = xy$ of a connected graph G is a cut edge of G if and only if e belongs to no cycle of G.

Or

(b) Show that the graph G with at least three vertices is 2-connected if and only if any two vertex of G lie on a common cycle.

18. (a) Prove that the simple graph is a tree if and only if any two distinct vertices are connected by a unique path.

Or

(b) Prove that a connected graph G is a tree if and only if every edge of G is a cut edge of G.

19. (a) Show that a matching M of a graph G is maximum if and only if G has no M — augmenting path.

Or

(b) Prove that for any graph G for which $\delta > 0, \alpha' + \beta' = n$.

20. (a) Let G be a simple graph with $n \geq 3$ vertices. If for every pair of nonadjacent vertices u, v of G, $d(u) + d(v) > n$, then prove that G is Hamiltonian.

Or

(b) If G is k-critical, then prove that $\delta(G) \geq K - 1$.

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PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 600 words.

21. (a) If the simple graphs G_1 and G_2 are isomorphic, then prove that $L(G_1)$ and $L(G_2)$ are isomorphic.

Or

(b) Prove that the set $Aut(G)$ of all automorphisms of a simple graph G is a group with respect to the composition of mappings as the group operation.

22. (a) Show that a ^{2}-connected graph G, any two longest cycles have at least two vertices in common.

Or

(b) State and prove *Whitney's* theorem.

23. (a) Prove that every tree has a center consisting of either a single vertex or two adjacent vertices.

Or

(b) If e is not a loop of a connected graph G, then prove that $\tau(G) = \tau(G - e) + \tau(G \circ e)$.

24. (a) If G is claw-free and connected graph of even order n, then prove that G is a 1 — *factor.*

Or

(b) State and prove *Tutte's* 1-factor theorem.

25. (a) Prove that for a nontrivial connected graph G, the following statements are equivalent

(i) G is eulerian

(ii) The degree of each vertex of G is an even positive integer

(iii) G is an edge-disjoint union of cycles.

Or

(b) Show that for any simple graph $G, 2\sqrt{n} \le \chi + \chi^c \le n+1$ and $n \le \chi\chi^c \le (\frac{n+1}{2})^2$.

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Reg. No. : .....

Code No. : 7395 **Sub. Code : WMAE 15/ VMAE 15**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2025.

First Semester

Mathematics

Elective II – ANALYTIC NUMBER THEORY

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($15 \times 1 = 15$ marks)

Answer ALL questions.

Choose the correct answer.

1. If $(a,b)=1$ then $(a^n,b^n)=1$ for all _____.

(a) $n \leq 1,\ k \leq 1$ (b) $n > 1, k > 1$

(c) $n \geq 1, k \geq 1$ (d) $n \geq 1, k \leq 1$

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2. If n^4+4 is composite then _____.

(a) $n=1$ (b) $n>1$

(c) $n<1$ (d) $n=0$

3. By well ordering principles every non empty set S of non-negative integers contains a _____.

(a) finite number (b) infinite number

(c) least element (d) greatest element

4. A real or complex valued function defined on the positive integers is called _____ function.

(a) Mobius (b) Dirichlet's

(c) Euler totient (d) arithmetical

5. $\phi(10)=$ _____.

(a) 1 (b) 4

(c) 2 (d) 6

6. _____ can be expressed as a product extended over the distinct prime divisors of n.

(a) $\mu(n)$

(b) $\phi(n)$

(c) Identify function $I(n)=\left[\frac{1}{n}\right]$

(d) Mobius function

7. Which of the following statement is not correct, if f is multiplicative?

(a) $f(1)=1$ (b) $f(n)=f(1)f(n)$

(c) f is identically zero (d) $f(n)\neq 0$

8. Which of the following is a multiplicative function?

(a) μ (b) F

(c) σ (d) all

9. A multiplicative function f is called completely multiplicative if _____ for all m, n.

(a) $f(mn)\neq f(m)f(n)$

(b) $f(mn)=f(m)f(n)$

(c) $f(mn)\leq f(m)f(n)$

(d) $f(mn)\geq f(m)f(n)$

10. If $f(x)$ is said to be asymptotic to $g(x)$ as $x\to\infty$ then _____.

(a) $\lim_{x\to\infty}\frac{f(x)}{g(x)}=0$ (b) $\lim_{x\to\infty}\frac{f(x)}{g(x)}=\infty$

(c) $\lim_{x\to\infty}\frac{g(x)}{f(x)}=0$ (d) $\lim_{x\to\infty}\frac{f(x)}{g(x)}=1$

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11. The determination of the infimum of all θ such that the error term is $O(x^{\theta})$ is an unsolved problem known as _____ problem.

(a) Euler's summation

(b) Lioville's function

(c) Dirichlet's divisor

(d) Chebyshev's

12. Two lattice points (a,b) and (m,n) are mutually visible iff _____ is relatively prime.

(a) $a-m$ (b) $b-n$

(c) $a+m, b-n$ (d) $a-m, b-n$

13. The series $\sum_{n=1}^{\infty} \frac{\mu(n)}{n}$ is _____.

(a) diverges

(b) converges

(c) Oscillates

(d) either converges or diverges

14. The step function $\Im(x)$ has a jump _____.

(a) $\log p$ at p (b) 1 at each prime p

(c) 0 at each p (d) $\frac{1}{\log p}$ at p

15. _____ formula to determine as asymptotic formula for $\log [x]!.F(x) = [x]$.

(a) Legendre's (b) Euler's summation

(c) Dirichlet's (d) Asymptotic

PART B — ($5 \times 4 = 20$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

16. (a) Prove that given any integers a and b with $a > 0$ there exist unique integers q and r such that $b = qa + r$, $0 \leq r < a$ if $a \neq b$ then r satisfies the stranger inequalities $0 < r < a$.

Or

(b) State and prove Euclid theorem.

17. (a) Show that Dirichlet's multiplication is commutative and associative.

Or

(b) If $n \geq 1$ then show that $\log n = \sum_{d/n} \wedge(d)$.

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18. (a) If f and g are multiplicative then prove that its Dirichlet's product $f*g$ is multiplicative.

Or

(b) State and prove generalized inversion formula.

19. (a) If f has a continuous derivative f' on the interval $[y,x]$ where $0<y<x$ then prove that $\sum_{y<n\le x} f(n)=\int_x^y f(t)dt+\int_y^x (t-[t]f'(t)dt+$ $f(x)([x]-x)-f(y)([y]-y)$.

Or

(b) For $x>1$ prove that $\sum_{n\le x}\varphi(n)=\frac{3}{\pi^2}x^2+O(x\log x)$ so the average order of $\varphi(n)$ is $\frac{3n}{\pi^2}$.

20. (a) Let $p(\alpha)$ denote the n^{th} prime. Then prove the following asymptotic relations are logically equivalent.

(i) $\lim_{n\to\infty}\frac{\pi(x)\log x}{x}=1$

(ii) $\lim_{n\to\infty}\frac{\pi(x)\log\pi(x)}{x}=1$.

Or

(b) For any arithmetical function $a(n)$ let $A(x)=\sum_{n\le x}a(n)$ where $A(x)=0$ if $x<1$. Assume f has a continuous derivative on the interval $[y,x]$ where $0<y<x$. Then prove that

$$\sum_{y<n\le x} a(n)f(n)=A(x)f(x)-A(y)f(y)-\int_y^x A(t)f'(t)\,dt$$

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either 'a' or 'b'.

Each answer should not exceed 500 words.

21. (a) State and prove Euclidean algorithm.

Or

(b) Prove that every positive integer $n>1$ can be expressed as a product of primes, this representation is unique, a part from the order in which the factor occur.

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22. (a) State and prove Mobius inversion formula.

Or

(b) Prove that:

(i) $\varphi(p^\alpha) = p^{\alpha-1}$ for prime p and $\alpha \geq 1$

(ii) $\varphi(m,n) = \varphi(m)\varphi(n)(d/\varphi(d))$ where $d = (m,n)$

(iii) $\varphi(m,n) = \varphi(m)\varphi(n)$ if $(m,n)=1$

(iv) a/b implies $\varphi(a)/\varphi(b)$

(v) $\varphi(n)$ is even for $n \geq 3$, more over, if n has r distinct odd prime factors, then $2^r/\varphi(n)$.

23. (a) Let f be multiplicative. Prove that f is completely multiplicative iff $f^{-1}(n) = \mu(n) f(n)$ for all $n \geq 1$.

Or

(b) Prove that

(i) for every $n \geq 1$ then $\sum_{d/n} \lambda(d) = \begin{cases} 0 & \text{if } n \text{ is a square} \\ 1 & \text{otherwise} \end{cases}$ also $\lambda^{-1}(n) = |\mu(n)|$ for all n.

(ii) If f is multiplicative function and F is defined by $F(n) = \sum_{d/n} f(d)$ then F is also multiplicative.

24. (a) State and prove Dirichlet's formula.

Or

(b) If $x \geq 1$ then prove the following :

(i) $$\sum_{n \leq x} \frac{1}{n} = \log x + C + O\left(\frac{1}{x}\right)$$

(ii) $$\sum_{n \leq x} \frac{1}{n^s} = \frac{x^{1-s}}{1-s} + \xi(s) + O(x^{-s}) \quad \text{if} \quad s > 1, \; s \neq 1$$

(iii) $$\sum_{n \leq x} \frac{1}{n^s} = O\left(x^{1-s}\right) \text{ if } s > 1$$

(iv) $$\sum_{n \leq x} n^\alpha = \frac{x^{\alpha+1}}{\alpha+1} + O(x^\alpha) \text{ if } \alpha \geq 0$$

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25. (a) Prove that the following relations are logically equivalent :

(i) $\lim_{n\to\infty} \frac{\pi(x)\log x}{x} = 1$

(ii) $\lim_{n\to\infty} \frac{\vartheta(x)}{x} = 1$

(iii) $\lim_{n\to\infty} \frac{\Psi(x)}{x} = 1$

Or

(b) State and prove Legendre's identify and also prove that for every $x \geq 1$ we have $[x]! = \prod_{p\leq x} p^{\alpha(p)}$ where the product is extended over all primes $\leq x$ and $\alpha(p) = \sum_{m=1}^{\infty} \left[\frac{x}{p^m}\right]$.

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