(8 pages)

Reg. No. : .....

Code No. : 5886 **Sub. Code : VMAC 41**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2026

Fourth Semester

Mathematics – Core

ADVANCED ALGEBRA – II

(For those who joined in July 2024-2025 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. Suppose that L, K , F are three fields in the relation $L \supset K \supset F$ and suppose further that $[L : F] = 28$. Which one of the following is possible?

   (a) $[K : F] = 3$ (b) $[L : F] = 5$

   (c) $[K : F] = 11$ (d) $[K : F] = 7$

2. The degree of $\sqrt{2}\sqrt{3}$ over Q is

   (a) 4 (b) 2

   (c) 6 (d) 5

3. If K is an extension of F and if $a \in K$ then $F(a)$ is

   (a) a subfield of K containing a

   (b) a smallest subfield of K containing both F and a

   (c) not necessarily a subfield of K

   (d) a smallest subfield of F containing both K and a

4. The degree of the splitting field of $x^4 + x^2 + 1$ over Q is

   (a) 2 (b) 4!

   (c) 4 (d) 1

5. The number of roots of the polynomial $x^2 + 1$ over the ring of real quaternions is

   (a) 2 (b) at least 3

   (c) 0 (d) 1

6. Let F be a field of characteristic 11. The derivative of the polynomial x^{11} is

   (a) 0 (b) x^{10}

   (c) $10x^{11}$ (d) 1

7. $\sigma \in G(K,F)$ if and only if

(a) $\sigma(a) = a$ for all a in K

(b) $\sigma(a) = a$ for all σ

(c) $\sigma(a) = 0$ for all a in F

(d) $\sigma(a) = 0$ for all a in K.

8. If K is the field of complex numbers and if F is the field of real numbers then the order of the group $G(K, F)$ is

(a) 1 (b) 2

(c) 3 (d) 4

9. A finite extension K is a normal extension of F if

(a) the fixed field of $G(K, F)$ is K

(b) the fixed field of $G(K, F)$ is F

(c) the fixed field of $G(K, F)$ is empty

(d) the fixed field of $G(K, F)$ is $\subseteq K$

10. Which one of the following is not possible for the order of a field?

(a) 2 (b) 9

(c) 46 (d) 49

11. The cyclotomic polynomial $\Phi_2(x)$ is

(a) $x-1$ (b) $x+1$

(c) x^2+x+1 (d) x^2+1

12. If the finite field F has p^m elements then every a in F satisfies

(a) $a^{p^m}-1=a$ (b) $a^{p^m}=0$

(c) $a^2=a$ (d) $a^{p^m}=a$

13. The only irreducible polynomials over the field of real numbers are of degree

(a) at least 3 (b) at most 3

(c) 1 or 2 (d) 1

14. If $x = 5 + 7i + 8j - 3k$ then $x + x^*$ is

(a) 5 (b) 10

(c) $10+14i+16j-6k$ (d) $5-7i-8j+3k$

15. If $x = 5 + 2i - j + k$ then $N(x)$ is.

(a) 31 (b) 29

(c) 10 (d) 7

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that elements in K which are algebraic over F form a subfield.

Or

(b) Show that algebraic over algebraic is algebraic.

17. (a) State and prove the Remainder theorem.

Or

(b) If $f(x) \in F[x]$ is irreducible and if characteristic of F is 0, prove that $f(x)$ has no multiple roots.

18. (a) Prove that the fixed field of G is a subfield of K.

Or

(b) If K is a finite extension of F, prove that $G(K, F)$ is a finite group and $\circ(G(K,F)) \leq [K:F]$.

19. (a) Let F be a finite field with q elements and suppose that $F \subset K$ where K is also a finite field. Prove that K has q^n elements for some integer n.

Or

(b) For every prime number p and every positive integer m prove that there exists a field having p^m elements.

20. (a) Let C be the field of complex numbers and suppose that the division ring D is algebraic over C, prove that $D = C$.

Or

(b) If $a \in H$, prove that $a^{-1} \in H$ is and only if $N(a) = 1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

21. (a) If L is a finite extension of K and if K is a finite extension of F prove that L. is a finite extension of F.

Or

(b) Exhibit a polynomial of degree 4 over Q satisfied by $\sqrt{2}+\sqrt{3}$. What is the degree of $\sqrt{2}+\sqrt{3}$ over Q? Prove your answer.

22. (a) If F is of characteristic 0 and if a, b are algebraic over F, prove that their exists an element c in $F(a, b)$ such that $F(a, b) = F(c)$.

Or

(b) Let $f(x) \in F[x]$ be of degree $n \geq 1$. Prove that there is an extension E of F of degree at most $n!$ in which $f(x)$ has n roots.

23. (a) If K is a field and $\sigma_1, \sigma_2, \cdots \sigma_n$ are distinct automorphisms of K, prove that it is impossible to find elements $a_1, a_2, \cdots a_n$, not all zero, in K such that $a_1 \sigma_1(u) + a_2\sigma_2(+) \cdots a_n\sigma_n(u) = 0$ for all u in K.

Or

(b) Prove that K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

24. (a) If F is a finite field and $\alpha \neq 0$, $\beta \neq 0$ are two elements of F. prove that we can find elements a and b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

Or

(b) Prove that every finite division ring is a field.

25. (a) State and prove the Frobenius theorem.

Or

(b) State and prove the Four square theorem.

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Code No. : 5887 **Sub. Code : VMAC 42**

M.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2026.

Fourth Semester

Mathematics — Core

FUNCTIONAL ANALYSIS

(For those who joined in July 2024-2025 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($15 \times 1 = 15$ marks)

Answer ALL questions.

Choose the correct answer :

1. Which one of the following is not true in a Banach space

   (a) norm is a continuous function

   (b) addition is jointly continuous

   (c) If M is a subspace of a normed linear space N then N/M is a normed linear space

   (d) closed unit sphere is always convex

2. Let T be a linear transformation. Which one of the following is possible?

(a) T is continuous but not bounded

(b) T is bounded but not continuous

(c) T is bounded and continuous

(d) T is continuous for all $x \neq 0$

3. If $p = \frac{3}{2}$, then $L_p{}^*$ is

(a) L_3 (b) $L_{\frac{2}{3}}$

(c) L_1 (d) $L_{\frac{1}{3}}$

4. Let T be linear transformation of B into B'. The graph of T is a subset of

(a) B (b) B'

(c) $B \times B'$ (d) $B' \times B$

5. Which one of the following is not true in a Hilbert Space?

(a) $\{0\}^{\perp} = H$

(b) $H^{\perp} = \{0\}$

(c) $S_1 \subseteq S_2 \Rightarrow S_1^{\perp} \subseteq S_2^{\perp}$

(d) $S^{\perp}$ is a closed linear subspace of H

6. Consider the following statements

(A) In a Hilbert space parallelogram law holds

(B) In $l_1^n (n > 1)$ parallelogram law holds. Then

(a) (A) is true but (B) is not true

(b) (B) is true but (A) is not true

(c) Both (A) and (B) are true

(d) Neither (A) nor (B) is true.

7. Which one of the following is not true?

(a) $T^{**} = T$

(b) $\|T^*\| = \|T\|$

(c) $(T_1 T_2)^* = T_1^* T_2^*$

(d) $(T_1 + T_2)^* = T_2^* + T_1^*$

8. An operator T is normal if and only if for all x

(a) $(T^* x, T^* x) = (Tx, x)$

(b) (Tx, x) is real

(c) $(Tx, Tx) = T^* x$

(d) $\|T^* x\| = \|Tx\|$

9. An operator P is a projection if

(a) $P^2 = P$

(b) $P^* = P$

(c) $P^{**} = P$

(d) $P^2 = P$ and $P^* = P$

10. If the dimension of H is 10, then the number of elements in the spectrum is

(a) 10

(b) more than 10

(c) at most 10

(d) any integer

11. An operator T is non-singular if and only if

(a) $T \neq 0$

(b) 0 is in the spectrum

(c) determinant of T is non zero

(d) $\det(T) = 0$

12. If $5 + 3i$ is an eigenvalue of T then an eigen value of T^* is

(a) $5 + 3i$ (b) 34

(c) $5 - 3i$ (d) 5 or 3

13. Let G be the regular elements and S be the singular elements of a Banach algebra, then

(a) G is closed and S is open

(b) both G and S are open

(c) Both G and S are closed

(d) G is open and S is closed

14. Any Banach algebra which is a division algebra equals

(a) C

(b) R

(c) Q

(d) set of integers

15. If $\sigma(x)$ is the spectrum of x then

(a) $\sigma(x)$ is not closed

(b) $\sigma(x)$ is non empty

(c) $\sigma(x)$ may be empty for some x

(d) $\sigma(x)$ is a subset of the Banach algebra A

PART B — ($5 \times 4 = 20$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

16. (a) Let T be a linear transformation of N into N'. Prove that T is continuous at the origin if and only if there exists a real number K such that $\|Tx\| \leq K\|x\|$ for every x in N.

Or

(b) If N is a normed linear space and x_0 is a non zero vector in N, prove that there exists a functional f_0 in N^* such that $f_0(x_0) = \|x_0\|$ and $\|f_0\| = 1$.

17. (a) State and prove the open mapping theorem without proving the lemma.

Or

(b) If M is a closed linear subspace of a Hilbert space H, prove that $H = M \oplus M^{\perp}$.

18. (a) If A_1 and A_2 are self adjoint operators on H, prove that their product A_1A_2 is self adjoint if and only if $A_1A_2 = A_2A_1$.

Or

(b) Prove that an operator T on H is unitary if and only if it is an isometric isomorphism of H onto itself.

19. (a) If P is the projection on a closed linear subspace M of H, prove that M is invariant under an operator T if and only if $TP = PTP$.

Or

(b) Define the determinant of an operator T on H and state any three properties of $\det(T)$

20. (a) Define the set of topological divisors of zero Z and show that Z is a subset of S, the set of singular elements.

Or

(b) Define the resolvent set of x and derive the resolvent equation.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

21. (a) Let M be a closed linear subspace of a normed linear space N. Prove that N/M is a normed linear space w.r.t. a suitable norm. Also show that N/M is a Banach space if N is a Banach space.

Or

(b) If N is a normed linear space, prove that the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak* topology.

22. (a) State and prove the uniform boundedness theorem.

Or

(b) State and prove Bessel's inequality.

23. (a) Let H be a Hilbert space and let f be an arbitrary functional in H^*. Prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H.

Or

(b) If T is an operator on H for which $(Tx, x) = 0$ for all x, prove that $T = 0$.

24. (a) If T is normal, prove that the M_i's are pair wise orthogonal and span H, where M_i is the eigenspace corresponding to the eigen value λi of T.

Or

(b) If P is a projection on H with range M and null space N, prove that $M \perp N$ if and only if P is self adjoint and in this case $N = M^{\perp}$.

25. (a) Prove that the mapping $x \to x^{-1}$ of G into G is continuous and is a homeomorphism of G onto itself, where G is the set of regular elements of a Banach algebra.

Or

(b) (i) If A is division algebra, prove that it equals the set of all scalar multiples of the identity.

(ii) If 0 is the only topological divisor of zero in A, prove that $A = C$.

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Code No. : 5420 **Sub. Code : WMASE 41/ VMASE 41**

M.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2026.

Fourth Semester

Mathematics

Skill Enhancement Course – III

INTRODUCTION TO MACHINE LEARNING AND APPLICATIONS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (15 × 1 = 15 marks)

Answer ALL questions.

Choose the correct answer :

1. _____ is not a simple process.
   (a) Machine learning (b) Cognitive learning
   (c) Programe (d) None

2. The field of AI dates back to the _____
   (a) 1940 (b) 1950
   (c) 1930 (d) 1960

3. _____ developed one of the earliest machine learning programs.
   (a) Euler (b) Charles Babbage
   (c) Arthur Lee Samuels (d) IBM

4. In big data, Data must be able to be verified based on both _____.
   (a) Accuracy and content
   (b) Verified and clear
   (c) Innovative and data points
   (d) None

5. Traditional BI tools typically analyze _____ of data rather than the entire data set.
   (a) Twitter (b) Face Book
   (c) Snapshots (d) Radio

6. _____ provides the foundation for building analytical models that deliver results you can trust.

(a) Hybrid cloud (b) Data refinement

(c) Both (a) and (b) (d) Neither (a) nor (b)

7. The role of analytics in an organization's operational processes has changed over the past _____.

(a) 30 years (b) 20 years

(c) 40 years (d) 10 years

8. There are _____ types of analytics.

(a) 3 (b) 4

(c) 2 (d) 1

9. _____ is the science of analyzing the data.

(a) Mathematics (b) Statistics

(c) Big data (d) None

10. _____ is used to solve a range of business problems.

(a) Mining tools (b) Data mining

(c) Market (d) Customer

11. _____ is the overall category that includes machine learning.

(a) BI (b) AI

(c) NLP (d) IBM

12. Machine reasoning helps make sense of _____.

(a) Connected data (b) Temperature

(c) Both (a) and (b) (d) None

13. When the data comes from finite set of values, it known as _____.

(a) Variables (b) Classification

(c) Correlation (d) Label

14. _____ is a specific method of machine learning.

(a) Neural network (b) Deep learning

(c) Both (a) and (b) (d) None

15. A neural network consists of _____ layers.

(a) One (b) Two

(c) Three (d) Three or more

PART B — (5 × 4 = 20 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

16. (a) What is machine learning?

Or

(b) What are the uses of machine learning?

17. (a) Write the four-shared characteristics in Big data?

Or

(b) What is the importance of the Hybrid cloud?

18. (a) How can the web experience be transformed to entire a customer to buy frequently?

Or

(b) What is the roles of statistics with machine learning?

19. (a) Explain Natural Language processing.

Or

(b) Explain unsupervised learning with example.

20. (a) What are the layers in Neural network?

Or

(b) How to understand machine learning technique?

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

21. (a) How should you think about machine learning?

Or

(b) Write about iterative learning from data.

22. (a) Explain Big data in content with machine learning.

Or

(b) Write the power of machine learning.

23. (a) What are the two types of approaches to advanced analytics? Explain.

Or

(b) Discuss the roles of Data mining with machine learning.

24. (a) What is the importance to understand the other subsets of AI?

Or

(b) Discuss the machine learning techniques.

25. (a) Write about Neural Networks.

Or

(b) Write a brief explanation in Deep learning.
