(8 pages)

Reg. No. : ....

Code No. : 20581 E **Sub. Code : EMMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2025.

Fourth Semester

Mathematics — Core

SEQUENCES AND SERIES

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $\lim_{n\to\infty} \frac{2n+1}{2n} =$ ________.

(a) 0 (b) 1

(c) 2 (d) –1

2. $\lim_{n\to\infty} \left(\frac{1+2+3+\ldots+n}{n^2}\right) =$ ________.

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) ∞

3. The following are true except ________. Let $(a_n) \to a$ and $(b_n) \to b$.

(a) $(a_n + b_n) \to a + b$ (b) $(a_n - b_n) \to a - b$

(c) $(a_n b_n) \to ab$ (d) $\left(\frac{a_n}{b_n}\right) \to \left(\frac{a}{b}\right)$

4. Example of a Cauchy sequence is ________.

(a) $\left(\frac{1}{n}\right)$ (b) (n)

(c) $((-1)^n)$ (d) (n^2)

5. Example of a series $\sum_{n=1}^{\infty} a_n$ which is divergent but $\lim_{n\to\infty} a_n = 0$

(a) $\sum_{n=1}^{\infty} \frac{1}{n}$ (b) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ (d) $\sum_{n=1}^{\infty} \frac{1}{n^3}$

6. If $a_n = \frac{n!}{n^n}$ then $\lim_{n\to\infty} \frac{a_n}{a_{n+1}} =$ ————.

(a) e (b) 1

(c) 0 (d) $\frac{1}{e}$

7. Applying Cauchy's root test the series $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$ is ————.

(a) convergent

(b) divergent

(c) neither convergent nor divergent

(d) both convergent and divergent

8. Test the convergence of the series $\sum \frac{2^n n!}{n^n}$

(a) converges

(b) diverges

(c) both (a) and (b)

(d) none

9. A series whose terms alternatively positive and negative is called an ————.

(a) alternating series

(b) convergence series

(c) divergence series

(d) non alternating series

10. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots = \sum (-1)^{n+1}\left(\frac{1}{n}\right)$ is an ———— series.

(a) alternating (b) convergence

(c) divergence (d) non alternating

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that $\lim_{n\to\infty} \frac{3n^2+2n+5}{6n^2+4n+7} = \frac{1}{2}$.

Or

(b) Show that the sequence $\left(1+\frac{1}{n}\right)^n$ converges.

12. (a) Let (a_n) be a Cauchy sequence. If (a_n) has a subsequence (a_n) converging to l, then $(a_n) \to l$.

Or

(b) Solve $\lim_{n\to\infty} \frac{n!}{n^n} = 0$.

13. (a) Discuss the convergence of the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \ldots$

Or

(b) Let $\sum a_n$ converge to a and $\sum b_n$ converge to b. Then $\sum (a_n \pm b_n)$ converge to $a \pm b$ and $\sum k a_n$ converge to ka.

14. (a) Test the convergence of the series $\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{2^2}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}+\frac{1}{3^3}\right)+\ldots.$

Or

(b) Show that $\lim_{n\to\infty}\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{n}-\log n\right)$ exists and lies between 0 and 1.

15. (a) Show that the following series converges :

$$\frac{1}{2^3}-\frac{1}{3^3}(1+2)+\frac{1}{4^3}(1+2+3)-\frac{1}{5^3}(1+2+3+4)+\ldots$$

Or

(b) Is the following series $\sum \frac{x^{n-1}}{(n-1)!}$ converges absolutely for all value of x. If yes show it.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If $(a_n) \to a$ and $(b_n) \to b$ then $(a_n + b_n) \to a + b$.

Or

(b) Show that $\lim_{n\to\infty} \frac{\log n}{n^p} = 0$ if $p > 0$.

17. (a) Prove that $\frac{1}{n}\left[(n+1)(n+2)\ldots(n+n)\right]^{1/n} \to \frac{4}{e}$.

Or

(b) Let (a_n) be a bounded sequence. Let $u_n =$ l.u.b of $\{a_n, a_{n+1}, \ldots\}$. Show that (u_n) is a monotonic decreasing sequence converging to $\lim\sup a_n$.

18. (a) Applying Cauchy's general principle of converge prove that $1-\frac{1}{2}+\frac{1}{3}-...+(-1)^n\frac{1}{n}+...$ is convergent.

Or

(b) (i) If $\sum c_n$ converges and if $\lim_{n\to\infty}\left(\frac{a_n}{c_n}\right)$ exists and is finite then $\sum a_n$ also converges.

(ii) If $\sum d_n$ diverges and if $\lim_{n\to\infty}\left(\frac{a_n}{d_n}\right)$ exists and is greater than zero then $\sum a_n$ diverges.

19. (a) Test the convergence of the series.

$$\frac{1}{3}x+\frac{1}{3}.\frac{2}{5}x^2+\frac{1}{3}.\frac{2}{5}.\frac{3}{7}x^3+...$$

Or

(b) If the following series $\sum\frac{n^3+a}{2^n+a}$ convergence. If yes then prove it.

20. (a) Show that the series $\sum(-1)^n\left[\sqrt{(n^2+1)}-n\right]$ is conditionally convergent.

Or

(b) Show that the series $\sum\frac{\sin n\theta}{n}$ converges for all values of θ.

(6 pages)

Reg. No. : ....

Code No. : 20582 E   Sub. Code : EMMA 42

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2025.

Fourth Semester

Mathematics — Core

FOURIER SERIES AND INTEGRAL TRANSFORMS

(For those who joined in July 2023 onwards)

Time : Three hours   Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The Fourier series of $f(x)$ in $(-\pi, \pi)$ is periodic with period

(a) 0   (b) π

(c) 2π (d) $-\pi$

2. Which one of the following is an odd function?

(a) $y = \cos x$ (b) $y = x^2$

(c) $y = |x|$ (d) $y = \sin x$

3. In the half-range cusine series for $f(x) = x^2$ in $(0, \pi)$ the value of a_0 is

(a) 0   (b) $\frac{2}{3}\pi^2$

(c) $\frac{3}{2}\pi^2$ (d) $\frac{4}{3}\pi^2$

4. If $y = f(x)$ is defined in $(0, 2\pi)$, then the root mean-square value of y is

(a) $\sqrt{\frac{1}{2\pi}\int_0^{2\pi} y^2 dx}$ (b) $\sqrt{\frac{1}{2l}\int_C^{2l} y^2 dx}$

(c) 0   (d) 1

5. Fourier transform of $f(x)$ is denoted by

(a) $f(s)$ (b) $\bar{f}(s)$

(c) $\bar{f}(x)$ (d) $F[f(s)]$

6. $F\{f(t)\} =$

(a) $L(t)$ (b) $L\{f(t)\}$

(c) $L\{\phi(t)\}$ (d) $\phi(t)$

7. $L(1) =$

(a) 1 (b) 0

(c) $\frac{1}{s}$ (d) $\frac{1}{s^2}$

8. $L(\cos t) =$

(a) $\frac{s}{s^2+1}$ (b) $\frac{s}{s+1}$

(c) $\frac{1}{s^2+1}$ (d) $\frac{1}{s+1}$

9. $L^{-1}\left(\frac{1}{s-a}\right) =$

(a) e^t (b) e^{at}

(c) t (d) t^n

10. $L^{-1}\left(\frac{a}{s^2-a^2}\right) =$

(a) $\cosh t$ (b) $\sinh t$

(c) $\cosh at$ (d) $\sinh at$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain the Fourier series of even functions.

Or

(b) Find the Fourier series of $f(x) = x^2$ in $(0, 2l)$.

12. (a) Find the half-range cosine series of $f(x) = x\sin x$ in $(0, \pi)$.

Or

(b) Derive the complex form of Fourier series.

13. (a) Find the Fourier transform of $e^{-a^2x^2}$.

Or

(b) State and prove convolution theorem.

14. (a) Find $L(\sin^2 2t)$.

Or

(b) Find $L(t^2 + 2t + 3)$.

15\. (a) Find $L^{-1}\left[\dfrac{s}{(s^2+a^2)^2}\right]$.

Or

(b) Find $L^{-1}\left[\dfrac{1}{s(s+1)(s+2)}\right]$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16\. (a) Find the Fourier series of period $2l$ for the function $f(x)=x(2l-x)$ in $(0,2l)$. Deduce the sum of $\dfrac{1}{1^2}-\dfrac{1}{2^2}+\dfrac{1}{3^2}-\ldots$

Or

(b) Find the Fourier series expansion for the function $f(x)=\cosh ax$ in $(-\pi,\pi)$.

17\. (a) Find the half-range cosine series of $f(x)=x(\pi-x)$ in $(0,\pi)$. Hence find the sum of the series $\dfrac{1}{1^4}+\dfrac{1}{2^4}+\dfrac{1}{3^4}+\ldots+\infty$.

Or

(b) State and prove Parseval's theorem.

18\. (a) State and prove Fourier integral theorem.

Or

(b) State and prove Parseval's identity.

19\. (a) Evaluate $\int_0^\infty te^{-3t}\sin t\,dt$.

Or

(b) Find $L\left(\dfrac{1-e^t}{t}\right)\Big/L\left(\dfrac{1-e^t}{t}\right)$.

20\. (a) Find $L^{-1}\left(\dfrac{(s-3)}{s^2+4s+13}\right)$.

Or

(b) Find $L^{-1}\dfrac{1}{(s+1)(s^2+2s+2)}$.

(7 pages) **Reg. No. :** ....

Code No. : 20589 E **Sub. Code : EEMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION,
APRIL 2025.

Fourth Semester

Mathematics

Elective — NUMERICAL METHODS

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. $\nabla =$ ______.

(a) $1+E$ (b) $1-E$

(c) $1+\Delta$ (d) $1-E^{-1}$

2. The algebraic sum of the errors in any column of the difference table is ______.

(a) 0 (b) E

(c) 1 (d) –1

3. The first divided difference $[x_0, x_1] =$ ______.

(a) $[x_1, x_2]$ (b) $-[x_1, x_0]$

(c) $[x_1, x_0]$ (d) $[x_{n-1}, x_n]$

4. ______ formula is the average of Gauss forward and backward formulae.

(a) Bessel's (b) Sterling's

(c) Newton's (d) Gauss

5. The error in the Trapezoidal rule is of order ______.

(a) h (b) h^2

(c) h^3 (d) h^4

6. The minima and maxima of a function can be obtained by equating ______ to zero.

(a) first derivative

(b) second derivative

(c) first divided difference

(d) second divided difference

7. ______ method is a step by step method.

(a) Taylor's (b) Picard's

(c) Euler's (d) Romberg's

8. Runge-Kutta method of first order is ________ method.

(a) Taylor's (b) Euler's

(c) Milne's (d) Modified Euler's

9. The method of refining an initially crude estimate by means of more accurate formulae is known as ________ method.

(a) Euler's (b) Runge-Kutta

(c) Taylor's (d) Predictor-corrector

10. Predictor formula is used to predict the value of ________ at x_{i+1}.

(a) y (b) $\frac{dy}{dx}$

(c) $\frac{d^2y}{dx^2}$ (d) x

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Form the difference table of the function $y = x^3 + x^2 - 2x + 1$, $x = -1, 0, 1, 2, 3, 4$.

Or

(b) Prove : $hD = \log(1+\Delta) = -\log(1-\nabla) = \sinh^{-1}(\mu\delta)$.

12. (a) For the following data find $f(9)$ using Newton's forward interpolation formula :

x :	8	10	12	14	16
$f(x)$:	1000	1900	3250	5400	8950

Or

(b) Using Stirling's formula, compute y_{35} given that $y_{10} = 600$, $y_{20} = 512$, $y_{30} = 439$, $y_{40} = 346$, $y_{50} = 243$.

13. (a) Derive a formula for $\frac{dy}{dx}$ at $x = x_0$ using Newton's forward difference formula.

Or

(b) Derive the Newton-Cote's quadrature formula.

14. (a) Solve the initial value problem $y' = 2y + 3e^x$, $y(0) = 0$ at $x = 0.2$ by Taylor's method.

Or

(b) Find the third approximation for the initial value problem $y' = 1 + y^2$, $y(0) = 0$ using Picard's method.

15. (a) Write a short note on Predictor-Corrector method.

Or

(b) Derive the Adams-Bashforth predictor formula.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Find the missing terms :

x:	0	1	2	3	4	5	6
y:	2	5	8	–	38	–	3

Or

(b) Prove that $\nabla^r f(r) = \Delta^r f(x-r)$ for any positive integer r.

17. (a) From the following data, estimate the population for the year 1945 :

Year :	1941	1951	1961	1971	1981	1991
Population :	2500	2800	3200	3700	4350	5225

Or

(b) Using Gauss's backward interpolation formula, estimate the value of $\sin 45^\circ$ from the following table :

x°:	20	30	40	50	60	70
$\sin x^\circ$:	0.342	0.502	0.642	0.766	0.866	0.939

18. (a) Obtain the value of $\log 2^{1/3}$ from $\int_0^1 \frac{x^2\,dx}{1+x^3}$ using Simpson's one-third rule with $h = 0.25$.

Or

(b) From the following data, evaluate $\left(\frac{d\theta}{dt}\right)_{t=0.6}$ and $\left(\frac{d^2\theta}{dt^2}\right)_{t=0.6}$:

t:	0	0.2	0.4	0.6	0.8	1.0
θ:	0	0.12	0.49	1.12	2.02	3.20

19. (a) Given $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$, $y(1) = 1$. Evaluate $y(1.3)$ by modified Euler's method.

Or

(b) Compute $y(0.1)$ and $y(0.2)$ by Runge-Kutta method of fourth order, given $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$.

20. (a) Find $y(0.4)$ by Milne's method for $y' = 1 + xy$, $y(0) = 2$.

Or

(b) Using Adams-Bashforth method, determine $y(1.4)$ given that $y' - x^2 y = x^2$, $y(1) = 1$.

(6 pages) **Reg. No. :** ....

Code No. : 20595 E **Sub. Code : ESMA 41**

B.Sc. (CBCS) DEGREE EXAMINATION, APRIL 2025.

Fourth Semester

Mathematics

Skill Enhancement Course

GEOGEBRA

(For those who joined in July 2023 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. GeoGebra is an interactive ———— system.
   (a) calculus (b) algebra
   (c) geometry (d) none of these

2. The extension of GeoGebra is
   (a) .gbb (b) .8gb
   (c) .bgg (d) .gbg

3. In GeoGebra, name a new object by typing
   (a) Name (b) name
   (c) name = (d) object =

4. Vectors are always named with ————.
   (a) Lower case letter
   (b) Upper case letter
   (c) Both (a) and (b)
   (d) Numbers

5. Export the graphics view to the clipboard using
   (a) Ctrl-C
   (b) Ctrl-Shift-C
   (c) Shift-C
   (d) Alt-C

6. Inserting pictures from the clipboard to OO writer using
   (a) Ctrl-V (b) Shift-V
   (c) Ctrl-P (d) Shift-P

7. In a GeoGebra window grid is in ———— menu.

(a) Insert (b) View

(c) Tool (d) File

8. In a spreadsheet view, the cell in column A and row 1 is named as

(a) 1A (b) A1

(c) Cell 1 (d) Cell A

9. dpi stands for

(a) dots per inch

(b) dot picture in

(c) dot picture idea

(d) dot picture insert

10. To export the figure as a dynamic worksheet using the menu

(a) View (b) File

(c) Insert (d) Tool

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 250 words.

11. (a) What is GeoGebra and how does it work?

Or

(b) How to save GeoGebra files? Explain.

12. (a) Explain the regular Hexagon Construction.

Or

(b) How to use sliders to modify parameters?

13. (a) Explain the parameters of a Linear equation.

Or

(b) Explain the exploring properties of reflection.

14. (a) How to insert static text? Explain.

Or

(b) How to attach text to an object? Explain.

15. (a) Explain the Best fit line.

Or

(b) Write a short note on GeoGebra tube.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

Each answer should not exceed 600 words.

16. (a) Explain the installation process of GeoGebra.

Or

(b) Explain the equilateral triangle construction.

17. (a) How to visualize the theorem of Tabs? Explain.

Or

(b) Explain the Library of functions.

18. (a) Explain exporting pictures to the clipboard.

Or

(b) Explain creating a Geometric figures memory game.

19. (a) Explain the rotation of a polygon.

Or

(b) Explain the visualizing a system of equations.

20. (a) Explain exploring basic statistics.

Or

(b) Explain creating dynamic worksheets.
